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Carbon Footprint for dynamically rebalanced portfolios.

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Abstract

The ambitious objectives stated in the latest Paris Agreement 2015 to limit the average global increase in temperature at the 2°C before preindustrial levels, and possibly to 1.5°C, will need tectonic shift in infrastructure, energy production, industrial processes and, down in the scale, to our lifestyles. As investors will play a key role to finance these objectives, they need a robust framework to measure the impact of their investments while monitoring long-term risk related to climate change. Derived from standard financial reporting, we propose such a framework to measure a complete carbon footprint for dynamically rebalanced funds. We enhance this with the derivation of carbon performance attribution of funds versus their benchmarks, both for absolute (ex. carbon emissions) and relative measures (ex. carbon intensity).

Key words: Carbon footprint, carbon performance attributions.

1 Introduction

With the recent Paris Agreement in 2015 (UN Treaties-XXVII, 2015), global consciousness of climate change risk has reached the status of a legally binding framework for countries to limit the "*increase in the global average temperature to* $2°C$ *above preindustrial level, and to pursue effort to limit such increase to* $1.5°C$." The Agreement's objective is a long-term global effort in the management of a public supernational good (the climate) and will have significant impacts on governments, policymakers, households and consumers, corporates and investors. Specifically for investors, climate change represents a threefold challenge: duty, risk and opportunity.
First of all, as per their role in societies, investors are asked to redirect significant investments (both directly, with equity investments and indirectly, via financing governments and companies through bonds) to meet the objectives of the Paris Agreement and other national or regional regulations already in place.
Second, climate change represents a nontrivial risk, from the increase of the likelihood of natural disasters to the impact on long-term economic growth. Indeed, climate change has a clear impact on nations' economic health, which in turn is transmitted to financial markets through the standard channels (resources and commodity prices, long-term debt, political risks, expectations). Understanding and managing these risks will be key for investors. For this, a large stream of literature has already pointed out how the extraordinary and unprecedented economic growth in the last 60 years is not sustainable in a Hicksian sense (see Hicks (1946), Rees (1992) and Dewan (2006) for further details). As a matter of fact, the new policies needed to tackle the climate change could likely have a significant impact on global economic paths. With a closer focus on the Energy sector, in particular oil and other fossil resources, there is a significant risk that part of the actual reserves cannot be burnt if we want to keep the trajectory of global temperature in line with the Paris Agreement. The Carbon Tracker Initiative (see CTI (2011)) reports that if the top 100 listed companies in the Coal and Oil&Gas sectors were to burn their reserves over the next 40 years, we would deviate significantly from the 2°C trajectory. Stevens (2016) also records similar conclusions. But for companies in this business, market prices and valuations strongly reflect their current reserves. If these reserves cannot be burnt, the financial impact will be huge and likely disruptive, causing systemic risk for all financial players. On this topic, see again CTI (2011) and the speech of Bank of England Governor Carney (Carney, 2015).
Third, this new framework is expected to initiate and reinforce tectonic shifts in capital allocation towards renewable energy production, sustainable industrial processes and efficient allocation of resources. The probable outcome will be, on the one hand, the emergence of new industries and corporates that will take advantage of new policies and environmental constraints while, on the other hand, noncompetitive industries could be negatively impacted. Opportunities will arise and investors would better integrate this dimension in order to detect new trends and allocate their investments accordingly. To face these challenges, investors need a precise measurement of the risk they face and their contributions to the objectives of a low carbon economy. Among the class of Eco-

logical Footprints measurements (Wackernagel and Rees, 1998), Carbon Footprint (CF) is a powerful tool that enables governments, households and corporates to measure their contribution to global warming as well as the efficiency of policies deployed to limit it. CF is universally admitted to stand for the total amount of greenhouse gas (GHG) emissions with potential global warming impact, i.e. it is usually measured in mass units (tons of CO_2 or equivalent) rather than area-based (the amount of land necessary to absorb the emissions). Notwithstanding the popularity CF enjoys in the media, in political debates and in the academia, there has been a significant work to converge toward a standardized definition of what a CF should be and how corporates, households and governments should calculate them. It is out of the scope of this paper to reproduce a complete review of the literature on this, so we refer to the ISO Guidelines (ISO 14067, 2013), the GRI Initiative, Wiedmann and Minx (2008), Singh et al (2009), Herva et al. (2011), Čuček et al. (2012) for further details. This paper does not focus on a specific measure of carbon emission. Our contribution is instead a unified framework for investors with dynamic allocations where they can measure the impact of their investments on global warming and transition to a low carbon economy. With respect to this approach, we use a generic collection of carbon measures, each of them capturing a specific side of a complete carbon footprint. Indeed, as pointed out by Galli et al. (2012), no single indicator is able to give a representative indication of the real impact, while a better picture can be drawn from a collection of indicators. Our framework is therefore flexible to the choice of measures each investor finds adapted to her investments. While we advocate for institutionalization and standardization of carbon footprints, we believe that the optimal choice of measure should depend on the type of investments. For example, as shareholders, equity investors can have an active role in the way corporates manage their climate change risk and the transition to a low carbon economy. On the other side, fixed income investors have very low impact on corporate management, but they can have significant impact if they favor climate change-related projects (impact investing). As such, the same indicator may not work for both categories of investors. Within our framework, investors will select a family of carbon measures as their inputs, according to their preferences and needs, and produce a global set of carbon footprints, each of them giving a representative and synthetic view of the portfolio's carbon risk. While not restrictive, the paper takes the point of view of an equity fund, but the same ideas can be adapted to fixed income funds. We define the *Absolute* and *Relative (Intensity)* contributions that are attributable to the fund, by taking into account both dynamic changes in the fund's compositions (rebalancings) and inflow/outflows. The framework is well adapted to carbon footprinting for ETFs and other Index funds, especially those that change their composition regularly (among other, trackers of Alternative Beta and Factor-based strategies). We also derive carbon footprint attribution that, in line with the standard performance attribution, should allow investors (and fund manager) to understand the main drivers of their carbon footprint. The application of our framework is shown on two practical examples.

Variable	GHG	Description
CB^1	Scope 1	GHG emissions generated from burning fossil fuels and production processes which are owned or controlled by the company
CB^2	Scope 2	GHG emissions from consumption of purchased electricity, heat or steam by the company
CB^3	Scope 3	Other indirect GHG emissions, both upstream and downstream, excluded by Scope 2
CB^4	Direct + First Tier Indirect	GHG Scope 1 emissions plus the GHG emissions of direct suppliers

Table 1: Measures of GHG emissions. All are expressed in ktCO_2e. *Source: TruCost, GHG Protocol.*

2 Data

Let F_t be the value (Asset Under Management) of a fund invested in equities whose weights within the portfolio, at time t, are denoted by w_t^F. We assume that B_t is a reference index to which the fund is benchmarked, and the fund solely consists of stock from the reference index. Stocks' weights in the reference index are denoted by w_t^B. We shall assume that the reference index weights are proportional to the total equities' market capitalizations adjusted by their free float, the standard for market indices like the Stoxx Europe 600 Index or the S&P 500 Index. B_t represents the total market value of the benchmark at time t and it is driven by the market fluctuations, while F_t moves according to market fluctuations and, from time to time, jumps following inflows/outflows activity from investors. By definition:

$$\forall t, i \qquad w_{t,i}^B = 0 \Rightarrow w_{t,i}^F = 0$$

Although not necessary, this assumption simplifies the analysis. For each stock in the reference index, we denote CB the vector of different carbon-related measures:

$$CB_{i,y} = \left(CB_{i,y}^1, CB_{i,y}^2, \ldots, CB_{i,y}^j\right)$$

where i indexes all the stocks in the reference index, y denotes the year on which the measures are accounted for, and j denotes all specific measures at disposal (for example, GHG emissions, Energy production and their sources, reserves). We denote by $R_{i,y}$ the revenue of company i for year y. Stocks' revenues are expressed in millions of the reference index currency. Table 1 describes some of the most used measures of GHG emissions. With any of these measures investors can easily obtain the standard carbon footprints. For practical purposes, we shall follow a variant of the definition given in Wiedmann and Minx (2008), specifically adapted to corporates:

The Carbon footprint is a measure of the exclusive total amount of greenhouse gases (GHG) emissions that is directly and indirectly upstream

caused by the business activity of the corporate.

With respect to the definition in Wiedmann and Minx (2008), we include all greenhouses gas, as defined by Kyoto Protocol, rather than carbon dioxide (CO_2) alone. Other gases are adjusted by a global warming coefficient and all is measured in thousands of tons of CO_2-equivalent (ktCO_2e). Furthermore, we attribute to each corporate the total amount of emissions directly attributable to its activities (Scope 1), their energy purchases (Scope 2) and other GHG emissions from its direct suppliers (upstream Scope 3). We believe that the addition of Scope 3 emissions is necessary to have a comprehensive view of the total carbon impact, although we are aware that this measure is often difficult to obtain (see, for example, Huang et al. (2009) and Downie and Stubbs (2013)). However, we do not include Scope 3 downstream emissions as they are usually not easily manageable by corporate managements. On the other side, corporate managements can take all the actions to limit carbon emissions and pressure their suppliers to do so. Table 2 collects the variables we use for energy producers in the reference index, and GHG emissions associated to energy production from fossil sources. Finally, Table 3 collects the variables we use to measure fossil reserves (both 1P and 2P) broken down according to their sources and the associated GHG emission. Carbon measures are usually disclosed in the companies' yearly statements, with some delay. The same is true for Revenues, although partial quarterly data may be available before. Therefore $CB_{i,y}$ is usually known with some delay during the course of year $y+1$. As the portfolio's weights may change over time due to regular rebalancing and as its market value can be impacted by inflows/outflows, fair accounting of carbon

Variable	Source
Renewable	
CB^5	Biomass
CB^6	Geothermal
CB^7	Hydroelectric
CB^8	Solar
CB^9	Wave and Tidal
CB^{10}	Wind
Fossil	
$CB^{11}, CB^{11,e}$	Coal
$CB^{12}, CB^{12,e}$	Liquefied Natural Gas
$CB^{13}, CB^{13,e}$	Liquefied Petroleum Gas
$CB^{14}, CB^{14,e}$	Natural Gas
$CB^{15}, CB^{15,e}$	Nuclear
$CB^{16}, CB^{16,e}$	Petroleum

Table 2: Measures of energies' production from different sources. The measures of production are expressed in GWh. For fossil sources we also have the measures of emissions linked to production, denoted by $CB^{11,e} \dots, CB^{16,e}$, expressed in kt$CO_2$e. *Source: TruCost, GHG Protocol.*

Variable	Source	Breakdown
CB^{17}	Coal	Metallurgical
CB^{18}	Coal	Other
CB^{19}	Coal	Thermal
CB^{20}	Gas	Natural
CB^{21}	Gas	Shale
CB^{22}	Oil	Conventional
CB^{23}	Oil	Unconventional

Table 3: Measures of energies' reserves. Metallurgical Coal and Oil are expressed in millions of barrels (MMbbl); Other and Thermal Coal in ktCO_2e while Gases are expressed in billions of cubic feet equivalent (bcf). The related GHG emissions are instead given in ktCO_2e. *Source: TruCost, GHG Protocol.*

emissions related to the fund would require daily data. This can be achieved by equally spreading yearly measures on each trading day: $CB^{j,daily}_{t,i} := CB^{j}_{i,y}/n(y)$, where Jan, $1^{st} \leq t \leq$ Dec, 31^{th} of year y and $n(y)$ is the number of trading days in the year. In what follows, we drop the daily notation as we consider daily data.

3 Absolute and Relative Contributions

Assume that on trading day t, the daily-adjusted carbon measure for company i is $CB^1_{t,i}$. We recall that following the definition given in Table 1, this measure corresponds to Scope 1 GHG emission in $ktCO_2e$. We define the proportion of these daily emissions attributable to fund F as the Fund's Daily Contribution $DC^F_t(CB^1)$. This can be calculated on an ownership-ratio basis: if $\theta^B_{t,i}$ is the total number of free floating shares available in the market and $\theta^F_{t,i}$ is the number of shares held by the fund then

$$\begin{aligned} DC^F_t(CB^1) &= \sum_{i=1}^{N} \frac{\theta^F_{t,i}}{\theta^B_{t,i}} CB^1_{t,i} \\ &= \sum_{i=1}^{N} \frac{\frac{F_t w^F_{t,i}}{PR_{t,i}}}{\frac{B_t w^B_{t,i}}{PR_{t,i}}} CB^1_{t,i} \\ &= \frac{F_t}{B_t} \sum_{i=1}^{N} \frac{w^F_{t,i}}{w^B_{t,i}} CB^1_{t,i} \end{aligned}$$

where the second equality comes from the definition of the weight, $PR_{t,i}$ is the price of stock i at time t in the reference index currency and N is the number of securities in the reference index. It should be noticed that, within this framework, we attribute all the emissions linked to the stocks to their shareholders only. First of all, it could be argued that debt holders should also be entitled to a proportion of the carbon emissions. An alternative could be to calculate the ownership ratio relatively to companies' assets rather than equities only. Second, it is widely accepted that when it comes to carbon accounting, the *shared responsibility* among producers and customers should prevail (on this, see for example Lenzen et al (2007)). In particular, it could be argued that, at least for special sectors (Utilities with public-related business or electricity producers for example) part of the emissions should not be accounted on the shareholders only. For the first point, it is possible in theory to use both debt and equity to build fair carbon footprints. However, this would require more data on the capital structure for each stock in the reference index. For the second one, although we agree in principle, we prefer using the simple method and then account all emissions to the shareholders as, for this point to be addressed, one would require subjective assumptions, which, in turn, would give subjective carbon footprints. In any case, investors usually monitor their carbon footprints over time to measure the impacts of their investments, and they benchmark their actions to the market portfolio. As we can reasonably think in relative terms, we believe that the simple accounting method is appropriate. Let now X be one of the 23 carbon measures CB^j detailed in Section 2 or the Revenue measure R. We define the *absolute contribution* A of the fund F relative to the measure X over the period $[T_1, T_2]$ as the sum of daily values of X attributable to the fund:

$$A^F(T_1,T_2,X) := \sum_{t=T_1}^{T_2} DC_t^F(X)$$
$$:= \sum_{t=T_1}^{T_2} \frac{F_t}{B_t} \sum_{i=1}^{N} \frac{w_{t,i}^F}{w_{t,i}^B} X_{t,i} \quad (3.1)$$

Throughout the rest of the document we will drop the time dependence of A when it is not misleading and write $A^F(X)$ as the absolute contribution of the fund for the measure X.
We can write A in vector form: if $OW_t^F := F_t/B_t$ represents the ratio of the fund's value over the total value of the reference index and INV^F is the matrix of investment ratios (i.e. the ratios of weights between the fund and the reference index):

$$INV_{t,i}^F = \begin{cases} \frac{w_{t,i}^F}{w_{t,i}^B} & \text{if} \quad w_{t,i}^B > 0 \\ 0 & \text{if} \quad w_{t,i}^B = 0 \end{cases}$$

then

$$A^F(X) = (OW^F)' * diag\left(INV^F * X'\right)$$

The *relative contribution or intensity* I^F of the fund for the measure X is defined as the ratio between the absolute contribution for the measure X and the absolute contribution for the measure $X = Revenue$:

$$I^F(X) := \frac{\sum_{t=T_1}^{T_2} \frac{F_t}{B_t} \sum_{i=1}^{N} \frac{w_{t,i}^F}{w_{t,i}^B} X_{t,i}}{\sum_{t=T_1}^{T_2} \frac{F_t}{B_t} \sum_{i=1}^{N} \frac{w_{t,i}^F}{w_{t,i}^B} R_{t,i}} \quad (3.2)$$

or in vector form

$$I^F(X) = \frac{A^F(X)}{A^F(R)} \quad (3.3)$$

For GHG emission measures (CB^j, $j = 1,\ldots,6$) Intensity I provides the amount of emissions for 1 million Revenue.

Remark 3.1. *When we look at the intensity on a given trading day ($t = T_1 = T_2$), from* (3.2) *we have*

$$I^F(X) = \frac{\sum_{i=1}^{N} \frac{w_{t,i}^F}{w_{t,i}^B} X_{t,i}}{\sum_{i=1}^{N} \frac{w_{t,i}^F}{w_{t,i}^B} R_{t,i}} = \sum_{i=1}^{N} \lambda_{t,i} \frac{X_{t,i}}{R_{t,i}},$$
$$\lambda_{t,i} := \frac{\frac{w_{t,i}^F}{w_{t,i}^B} R_{t,i}}{\sum_{i=1}^{N} \frac{w_{t,i}^F}{w_{t,i}^B} R_{t,i}}$$

We remark that X/R is the stock's intensity measure while λ_i is the fraction of fund's revenue attributable to stock i. In other words, the instantaneous fund's intensity is given by the weighted average of its stocks' intensities, where the weights are given by the fraction of the fund's revenue attributable to each stock. We highlight that weighted averages of stocks' intensities, where the weights are the ones in the fund as opposed to the revenues, will give a biased measure on the fund's intensity.

4 Carbon attributions

As in a standard Brinson-like performance attribution (Brinson and Fachler, 1985; Brinson et al., 1986), we propose a simple framework where the difference in absolute (resp. intensity) measures A (resp. I) between the fund and the benchmark can be broken down into user-given categories. For this, let us assume that all stocks in the universe can be classified according to a given categorical scheme S, as for example industrial sectors or countries. We can also consider the case of dynamic categorical schemes, meaning that a stock can belong to different categories at different times. Typical examples are the categorical schemes based on companies' size, valuation, volatility or any other dynamic measure. To simplify the notation we will assume that the categorical scheme S is indeed static, although the method applies for dynamic schemes too. For the sake of simplicity, we will also denote each category as a *sector*. The goal of the attribution is to study the difference between the fund's carbon footprint and that of the reference index through the prism of Allocation and Selection Effect within each sector. In what follow, we denote $|S|$ as the number of sectors in the scheme S and the notation $i \in S_k$ means that stock i belongs to the sector S_k.

The Natural Benchmark Let X be one of the carbon measures introduced in Section2 and assume that, over a given period, the fund has significantly outperformed the benchmark. The fund could then have a higher measure $A^F(X)$ simply because its market value is higher than the benchmark. More generally, we recognize that the difference in financial performances between the fund and the benchmark could imply higher measures $A^F(X)$. In line with our fair accounting approach, we propose a specific benchmark that neutralizes differences between the fund market value and the reference index market values (i.e. financial performance differences).

For this, we introduce a family of theoretical funds, denoted by $(BF)_h$, $T_1 \leq h \leq T_2$, such that at time t the *t-th* fund of the family has the same market value of the fund F: $(BF_t)_t = F_t$ and $w_{t,i}^{BF_t} = w_{t,i}^B$. Basically the *t-th* fund of the family BF invests, on day t, the amount F_t according to the weights in the reference index at time t. We call BF the **natural benchmark** of F. The use of the natural benchmark allows us to neutralize the differences in carbon footprints arising from differences in performances of the fund with respect to the benchmark.

Absolute Measures From (3.1) we can write

$$A^F(X) = \sum_{t=T_1}^{T_2} \frac{F_t}{B_t} \sum_{i=1}^{N} \frac{w_{t,i}^F}{w_{t,i}^B} X_{t,i}$$
$$= \sum_{t=T_1}^{T_2} \frac{F_t}{B_t} \sum_{k=1}^{|S|} \sum_{i \in S_k} \frac{w_{t,i}^F}{w_{t,i}^B} X_{t,i}$$

If $W_t^{S_k,F} := \sum_{i \in S_k} w_{t,i}^F$ is the weight of the sector S_k in the fund and $w_{t,i}^F / W_t^{S_k,F}$ is the relative weight of each stock within its own sector then

$$A^F(X) = \sum_{t=T_1}^{T_2} \sum_{k=1}^{|S|} W_t^{S_k,F} \left(\frac{F_t}{B_t} \sum_{i \in S_k} \frac{w_{t,i}^F}{W_t^{S_k,F}} \frac{1}{w_{t,i}^B} X_{t,i} \right)$$

If we denote by $F_{k,t}$ a fund with the same market value of F at time t ($(F_{k,t})_t = F_t$) but invested only in stocks of sector S_k with the same weighting scheme (basically by taking initial weights w^F and scale it up to sum to 1), then from (3.1) we can write

$$\frac{F_t}{B_t} \sum_{i \in S_k} \frac{w_{t,i}^F}{W_t^{S_k,F}} \frac{1}{w_{t,i}^B} X_{t,i} = A^{F_{k,t}}(t,t,X)$$

so that finally

$$A^F(X) = \sum_{t=T_1}^{T_2} \sum_{k=1}^{|S|} W_t^{S_k,F} A^{F_{k,t}}(t,t,X) \quad (4.1)$$

Equation 4.1 simply states that the absolute contribution of the fund for the measure X is the sum of instantaneous weighted average of the sectors contributions. The same holds true for the natural benchmark BF:

$$A^{BF}(X) = \sum_{t=T_1}^{T_2} \sum_{k=1}^{|S|} W_t^{S_k,BF} A^{BF_{k,t}}(t,t,X)$$

where BF_k is a fund which invests the same value of the fund at time t (F_t) according to the reference index weights $w^B_{t,i}$. The excess contribution $E(X)$ of the fund's over its natural benchmark is then

$$E(X) = A^F(X) - A^{BF}(X) \tag{4.2}$$

For example, if $X = CB^1$, then $E(X) > 0$ means that the Scope 1 GHG emissions attributable to the fund are higher than the emission attributable to its natural benchmark. Since

$$E(X) := \sum_{t=T_1}^{T_2} E_t(X) \tag{4.3}$$

where

$$E_t(X) := \sum_{k=1}^{|S|} W_t^{S_k,F} A^{F_{k,t}}(t,t,X) - \sum_{k=1}^{|S|} W_t^{S_k,BF} A^{BF_{k,t}}(t,t,X) \tag{4.4}$$

we can concentrate on $E_t(X)$ and then add up all daily excess contributions to obtain the total excess contribution. As for the standard Brinson's performance attribution, equation 4.5 decomposes the daily excess contribution into three separate effects. The Allocation Effect measures the ability of the strategy to underweight (overweight) sectors with higher (lower) absolute contributions than the natural benchmark. Despite the standard Brinson performance attribution, here we look at negative contributions rather than positive.

$$\begin{aligned} E_t(X) = & \underbrace{\sum_{k=1}^{|S|} \left(W_t^{S_k,F} - W_t^{S_k,BF}\right)\left(A^{BF_{k,t}}(t,t,X) - A^{BF}(t,t,X)\right)}_{\text{Allocation Effect}} \\ & + \underbrace{\sum_{k=1}^{|S|} W_t^{S_k,BF}\left(A^{F_{k,t}}(t,t,X) - A^{BF_{k,t}}(t,t,X)\right)}_{\text{Selection Effect}} \\ & + \underbrace{\sum_{k=1}^{|S|} \left(W_t^{S_k,F} - W_t^{S_k,BF}\right)\left(A^{F_{k,t}}(t,t,X) - A^{BF_{k,t}}(t,t,X)\right)}_{\text{Interaction Effect}} \end{aligned} \tag{4.5}$$

The Selection Effect measures the ability of the strategy to select, among each sector, companies that make the final absolute contribution lower than the benchmark. The Interaction Effect measures the ability of the strategy to underweight (overweight) the sectors with higher (lower) absolute contributions than the sectors in the natural benchmark. Equation 4.5 highlights the importance of the choice of the natural benchmark in the carbon attribution: since $(F_{k,t})_t = F_t$, $(BF_{k,t})_t = F_t$ and $BF_t = F_t$, the instantaneous Allocation, Selection and Interaction Effects will not depend on the potential differences between the fund's and reference index's market values. Without such adjustment, these effects would have been biased by the difference in market value, making the instantaneous portfolio's carbon attribution positive (negative) if its market value is lower (higher), all other things being equals.

From (3.3) we have

$$\begin{aligned} EI(X) :=& \frac{A^F(X)}{A^F(R)} - \frac{A^{BF}(X)}{A^{BF}(R)} \\ =& \frac{A^{BF}(R) * \left(A^F(X) - A^{BF}(X)\right)}{A^F(R)A^{BF}(R)} \\ &+ A^{BF}(X)\frac{\left(A^{BF}(R) - A^F(R)\right)}{A^F(R)A^{BF}(R)} \end{aligned}$$

and from (4.2)–(4.3)

$$\begin{aligned} EI(X) =& \frac{1}{A^F(R)}\left(E(X) - I^{BF}(X)E(R)\right) \\ =& \frac{1}{A^F(R)}\sum_{t=T_1}^{T_2} EI_t(X) \qquad (4.6) \\ EI_t(X) :=& E_t(X) - I^{BF}(X)E_t(R) \end{aligned}$$

Intensity Measures The decomposition for Intensity measures is slightly different. Finally, from (4.5), we can break down the instantaneous excess intensity as follows:

$$\begin{aligned} EI_t(X) =& \underbrace{\sum_{k=1}^{|S|}\left(W_t^{S_k,F} - W_t^{S_k,BF}\right)\left(A^{BF_{k,t}}(t,t,X) - A^{BF}(t,t,X)\right)}_{\text{X-Allocation Effect}} \\ & \underbrace{- I^{BF}(X)\sum_{k=1}^{|S|}\left(W_t^{S_k,F} - W_t^{S_k,BF}\right)\left(A^{BF_{k,t}}(t,t,R) - A^{BF}(t,t,R)\right)}_{\text{R-Allocation Effect}} \\ & \underbrace{+ \sum_{k=1}^{|S|} W_t^{S_k,BF}\left(A^{F_{k,t}}(t,t,X) - A^{BF_{k,t}}(t,t,X)\right)}_{\text{X-Selection Effect}} \qquad (4.7) \\ & \underbrace{- I^{BF}(X)\sum_{k=1}^{|S|} W_t^{S_k,BF}\left(A^{F_{k,t}}(t,t,R) - A^{BF_{k,t}}(t,t,R)\right)}_{\text{R-Selection Effect}} \\ & \underbrace{+ \sum_{k=1}^{|S|}\left(W_t^{S_k,F} - W_t^{S_k,BF}\right)\left(A^{F_{k,t}}(t,t,X) - A^{BF_{k,t}}(t,t,X)\right)}_{\text{Interaction Effect}} \\ & \underbrace{- I^B F(X)\sum_{k=1}^{|S|}\left(W_t^{S_k,F} - W_t^{S_k,BF}\right)\left(A^{F_{k,t}}(t,t,R) - A^{BF_{k,t}}(t,t,R)\right)}_{\text{Interaction Effect}} \end{aligned}$$

For example, if $X = CB^1$, then $EI(X) > 0$ means that the Scope 1 GHG emissions' intensity of the fund is higher than the emissions' intensity of the natural benchmark. In other words, the fund is less carbon efficient since it is accountable for more GHG emissions per million of revenue than its natural benchmark. We see that high differences in the absolute contributions between the fund and the natural benchmark $E(X)$ tend to increase intensity differences, all other things being equal. On the other side, high differences in the absolute revenue $E(R)$ tend to decrease intensity differences, all other things being equal. In particular, $EI(X) = 0$ implies $E(X) = I^{BF}(X)E(R)$ so that for the fund to have the same intensity as the natural benchmark, the excess absolute contribution must grow linearly with the excess in revenue, the constant being given by the benchmark intensity. The Allocation and Selection terms above have the same interpretation as for the absolute measure attribution. But this time we have to consider both the effect of the carbon measure X and the Revenue R. It should be noted that the Revenue effects (Allocation and Selection) have negative impacts on the intensity, meaning that their increases tend to lower the total excess intensity, which in turns translate into a lower carbon intensity for the fund compared to the benchmark. We remark that in the decomposition (4.6) we still have the total fund's revenue $A^F(R)$. In Appendix A we propose an alternative decomposition that does not make reference to this quantity.

5 Applications

In this section we propose two simple applications of the carbon analysis developed in Sections 3–4. For both applications, the reference index is the Stoxx Europe 600 Index. The period of study goes from December, 30, 2005 to December, 30, 2016. Carbon measures are taken from Table 1 and we also introduce the followings:

$$\text{Energy production from fossil sources} := \sum_{i=11}^{16} CB^i \text{ in GWh}$$

$$\text{GHG emissions from fossil sources} := \sum_{i=11}^{16} CB^{i,e} \text{ in ktCO}_2\text{e}$$

$$\text{Green energy production} := \sum_{i=5}^{10} CB^i \text{ in GWh}$$

The size effect. We consider two portfolios invested in the largest and the smallest tiers of the investment universe. More precisely, at each rebalancing date (third Friday of March, June, September and December) the first portfolio invests in the top tiers stocks of the reference index. We denote this portfolio as *Large*. Similarly, the *Small* portfolio will invest in the small tiers stocks of the reference index. Stocks are weighted as in the reference index (scaled up so that they add to 1). Outside from

	Bench.	Large
GHG Direct plus First-Tier Indirect	8.46	7.7
GHG Scope 1	5.50	4.99
GHG Scope 2	0.79	0.71
GHG Scope 3	4.83	4.42
Energy production from fossil sources	0.82	0.49
GHG emissions from fossil sources	0.39	0.30
Green energy production	0.23	0.17
Revenue	18.08	16.21
GHG emission from reserves	45.75	55.78
Intensity Scope 1	0.30	0.31
Intensity Scope 2	0.04	0.04
Intensity Scope 3	0.27	0.27
Intensity Direct plus First-Tier Indirect	0.47	0.48

Table 4: Carbon Measures associated to the portfolio *Large* and its natural benchmark. GHG emissions are measured in $ktCO_2e$; revenues in millions of EUR, intensities in $ktCO_2e$/mEUR. Source DataStream, Stoxx, TruCost.

rebalancing dates, stocks that are removed from the reference index are also removed from the two portfolios and their weights are distributed over the other stocks on a pro-rata basis. Portfolios are calculated in EUR. Finally, dividends are reinvested in the index according to the Stoxx methodology. We assume that two investors are buying-and-holding these two portfolios for 1,000,000 EUR at inception (December, 30, 2005). Tables 4–5 give an overview of the carbon performance of the hypothetical investments into the portfolios *Large* and *Small* compared to their natural benchmarks. It should be noted that since the portfolios *Large* and *Small* do not have the same financial return over the period, it follows that their natural benchmarks are different. From Table 4, we have that the GHG Direct plus First-Tier Indirect emissions associated with the portfolio *Large* are 0.76$ktCO_2e$ lower than the ones of its natural benchmark. However the Revenue associated with the natural benchmark is higher (18.08 versus 16.21 million EUR) so that all Intensity measures for the natural benchmark and the portfolio *Large* are very similar. The portfolio *Large* is accountable for almost 10 $ktCO_2e$ more GHG emissions from reserves than the natural benchmark (55.78 versus 45.75 $ktCO_2e$). The portfolio *Small* instead is responsible for significantly higher carbon emissions (15.91 $ktCO_2e$ versus 9.31 $ktCO_2e$ for its natural benchmark when we look at the GHG Direct plus First-Tier Indirect Emissions). Although emissions are higher, its intensi-

	Bench.	Small
GHG Direct plus First-Tier Indirect	9.31	15.91
GHG Scope 1	6.04	10.63
GHG Scope 2	0.88	1.3
GHG Scope 3	5.32	8.58
Energy production from fossil sources	0.98	1.72
GHG emissions from fossil sources	0.46	0.53
Green energy production	0.27	0.43
Revenue	20.07	33.54
GHG emission from reserves	53.48	6.64
Intensity Scope 1	0.3	0.32
Intensity Scope 2	0.04	0.04
Intensity Scope 3	0.27	0.26
Intensity Direct plus First-Tier Indirect	0.46	0.47

Table 5: Carbon Measures associated to the portfolio *Small* and its natural benchmark. GHG emissions are measured in $ktCO_2e$; revenues in millions of EUR, intensities in $ktCO_2e$/mEUR. Source DataStream, Stoxx, TruCost.

ties are very similar to the natural benchmark (and the portfolio *Large* too), given the higher proportion of Revenue associated with this portfolio. It is interesting to notice that the GHG emissions from reserves are almost ten times lower than the natural benchmark since these reserves are often associated with Oil & Gas companies that, usually, rank among the biggest capitalization of the Reference Index. Tables 6–7 show the carbon attributions of the portfolios *Large* and *Small* with respect to their natural benchmarks when we consider the GHG Direct plus First-Tier Indirect Emissions. The first four columns (Allocation, Selection, Interaction and Total) contain the different components given in (4.4)–(4.5). The others columns show the values given in (4.1): the average sector weights, the cumulated GHG emission of each sector and their product (contribution), for both the portfolios and their natural benchmarks. For the portfolio *Large*, we see that the difference in GHG emissions (-0.76 ktCO_2e) is largely explained by a good Selection Effect (-0.59 ktCO_2e) and Allocation Effect (-0.19 ktCO_2e). More in details, the portfolio *Large* has a good Selection Effect in the Basic Material and Consumer Services sectors, signaling that large companies in these sectors tend to have lower GHG emissions compared to the natural benchmark. On the other side, the portfolio has a bad Selection Effect in the Industrial sector, which is consistent with the fact that such large industries may have larger GHG emissions. For the portfolio *Small* (Table 7), we know that the difference in GHG emissions is positive (6.6 ktCO_2e) and it is mainly explained by negative Selection Effect (9.3 ktCO_2e) with a marginal good Interaction Effect (-2.09 ktCO_2e) and Allocation Effect (-0.69 ktCO_2e). This is due to poor Selection Effect in the Basic Materials, Consumer Good and Utilities sectors, which in turns prove that small companies in these sectors tend to have larger GHG emissions. The GHG emission contribution of the Industrial sector in particular is quite high (3.43 ktCO_2e) compared to the equivalent in the natural benchmark (1.56 ktCO_2e), although the absolute emission of the sector in the portfolio and the natural benchmark is similar (14.68 ktCO_2e versus 13.43 ktCO_2e). This translates into a negative Allocation Effect with a very small Selection Effect. Said otherwise, Industrials may have larger GHG emission than the benchmark, but the portfolio *Small* amplifies this fact because usually Industrial stocks have smaller capitalizations than other sectors' stocks, meaning that the portfolio is finally overweighted on this sector. It is noticeable that the Utilities sector is by far the biggest absolute GHG contributor, for both the portfolio *Large* (43.11 ktCO_2e for 5.33% average weight) and in the *Small* (126.5 ktCO_2e for 2.11% average weight). Indeed, sectors in the portfolio *Small* have higher GHG emissions than *Large*, even if, except for Basic Materials, the differences are less pronounced. From this simple example we can conclude that, across sectors, company's size has a direct impact in the total GHG emissions. Said otherwise, smaller companies tend to have higher GHG emissions than their large counterparts, although this is not uniform within sectors. Finally, we notice that the single sector emissions in the natural benchmark for the portfolio *Large* and *Small* (columns GHG Emissions in Tables 6-7) are not the same. This is due to the fact that the portfolios do not have the same financial performance.

Large

Sector	Allocation	Selection	Interaction	Total	Avg. Weight *Large*	GHG Emissions	Contr.	Avg. Weight Bench.	GHG Emissions	Contr.
Total	-0.19	-0.59	0.01	-0.76		7.7			8.46	
Basic Materials	-0.01	-0.27	0	-0.28	8.14%	17.83	1.43	8.22%	21.12	1.71
Cons. Goods	-0.05	-0.17	-0.01	-0.22	16.31%	3.5	0.54	15.26%	4.73	0.68
Cons. Services	0.04	-0.27	0.05	-0.17	5.75%	1.83	0.1	7.16%	5.57	0.4
Financials	-0.05	-0.09	0	-0.14	24.52%	0.25	0.06	23.98%	0.65	0.14
Health Care	-0.09	-0.01	0	-0.09	11.55%	0.37	0.04	10.46%	0.43	0.04
Industrials	-0.09	0.22	-0.05	0.08	9.33%	14.18	1.29	11.86%	12.36	1.42
Oil - Gas	0.09	0.07	0	0.16	9.54%	21.15	1.97	8.74%	20.4	1.74
Technology	0.02	-0.01	0	0.01	3.13%	0.52	0.02	3.38%	0.69	0.02
Telecom.	-0.05	0	0	-0.05	6.36%	0.74	0.05	5.71%	0.74	0.04
Utilities	0	-0.08	0.01	-0.07	5.33%	43.11	2.2	5.24%	45.04	2.25

Table 6: GHG emission attributions for the portfolio *Large* over its natural benchmark. Source DataStream, Stoxx, TruCost.

Small

Sector	Allocation	Selection	Interaction	Total	Avg. Weight *Small*	GHG Emissions	Contr.	Avg. Weight Bench.	GHG Emissions	Contr.
Total	-0.69	9.37	-2.09	6.6		15.91			9.31	
Basic Materials	-0.15	2.55	-0.35	2.04	6.96%	53.3	3.81	8.22%	23.21	1.88
Cons. Goods	0.26	1.99	-0.57	1.68	9.86%	19.28	1.94	15.26%	5.15	0.75
Cons. Services	-0.24	0.61	0.67	1.04	14.92%	14.75	2.19	7.16%	6.12	0.44
Financials	-0.09	0.33	0.06	0.29	25.53%	2.16	0.58	23.98%	0.72	0.16
Health Care	0.4	0.07	-0.03	0.44	5.83%	1.13	0.06	10.46%	0.47	0.05
Industrials	0.49	0.06	0.33	0.88	22.38%	14.68	3.43	11.86%	13.43	1.56
Oil - Gas	-0.38	0.37	-0.22	-0.23	5.43%	26.19	1.38	8.74%	22.59	1.91
Technology	-0.11	0.02	0	-0.08	4.58%	1.28	0.06	3.38%	0.75	0.03
Telecom.	0.29	0.03	-0.02	0.3	2.37%	1.31	0.03	5.71%	0.82	0.05
Utilities	-1.15	3.34	-1.95	0.24	2.11%	126.5	2.43	5.24%	50.28	2.49

Table 7: GHG emission attributions for the portfolio *Small* over its natural benchmark. Source DataStream, Stoxx, TruCost.

Euro zone versus rest of Europe Our second example considers two portfolios invested respectively in all the stocks in the Euro Zone, and in the rest of Europe (Switzerland, UK, Sweden, Norway, Denmark, Iceland and Czech Republic) from the reference index (Stoxx Europe 600 Index). Portfolios are rebalanced quarterly (third Friday of March, June, September and December) and stocks are weighted as in the reference index (with a scaling factor so that final weights sum to 1). We denote these portfolios as *Euro* and *Ex-Euro*. Portfolios are calculated in EUR and their maintenance is the same as before. Again, we assume that 1,000,000 EUR is invested at inception on both portfolios. From Table 8 we see that the portfolio *Euro* is characterized by higher GHG emissions, especially Scope 1 emissions, which in turns translates into higher Direct + First-Tiers indirect emissions. Companies in the Euro zone tend to be higher GHG emitters than the natural benchmark, but their emissions are linked to their industrial processes rather than upstream or downstream GHG emissions. Indeed they have almost the double of the Energy production from fossil sources (Table 8), and relative double GHG emissions from fossil sources than the natural benchmark. They also have higher Green energy production. Finally, the revenues associated with the Euro zone companies are higher than the natural benchmark, but this is not enough to compensate for higher GHG emissions,

	Bench.	Euro
GHG Direct plus First-Tier Indirect	8.13	11.13
GHG Scope 1	5.29	7.88
GHG Scope 2	0.76	0.88
GHG Scope 3	4.62	5.66
Energy production from fossil sources	0.75	1.46
GHG emissions from fossil sources	0.35	0.67
Green energy production	0.21	0.4
Revenue	17.18	22.21
GHG Emission From Reserves	41.46	14.41
Intensity Scope 1	0.31	0.35
Intensity Scope 2	0.04	0.04
Intensity Scope 3	0.27	0.26
Intensity Direct plus First-Tier Indirect	0.47	0.5

Table 8: Carbon Measures associated to the portfolio *Euro* and its natural benchmark. GHG emissions are measured in ktCO_2e; revenues in millions of EUR, intensisties in ktCO_2e/mEUR. Source DataStream, Stoxx, TruCost.

	Bench.	Ex-Euro
GHG Direct plus First-Tier Indirect	9.16	6.02
GHG Scope 1	5.93	3.23
GHG Scope 2	0.87	0.74
GHG Scope 3	5.26	4.17
Energy production from fossil sources	0.97	0.16
GHG emissions from fossil sources	0.46	0.1
Green energy production	0.27	0.05
Revenue	19.86	14.58
GHG Emission From Reserves	53.8	84.15
Intensity Scope 1	0.3	0.22
Intensity Scope 2	0.04	0.05
Intensity Scope 3	0.26	0.29
Intensity Direct plus First-Tier Indirect	0.46	0.41

Table 9: Carbon Measures associated to the portfolio *Ex-Euro* and its natural benchmark. GHG emissions are measured in ktCO_2e; revenues in millions of EUR, intensisties in ktCO_2e/mEUR. Source DataStream, Stoxx, TruCost.

leading then to higher intensities: for the Direct + First-Tiers Indirect, the portfolio *Euro* produces 0.5 ktCO_2e par one million of EUR of revenue, while this number is at 0.47 ktCO_2e for the natural benchmark. The portfolio *Ex-Euro* is instead characterized by lower GHG emissions than its natural benchmark, as it is shown in Table 9. These companies are usually responsible for lower level of energy production and emission from fossil sources, but also lower levels of renewable green energy production. The share of revenue associated with *Ex-Euro* is also lower than its natural benchmark (14.58 versus 19.86 million EUR). Overall, its intensity is lower than the natural benchmark (0.41 ktCO_2e against 0.46 ktCO_2e for its natural benchmark).

Table 10 reproduces the carbon emission attribution detailed in equations (4.4)–(4.5) for the portfolio *Euro* against its natural benchmark when we consider the GHG Direct + First-Tier Indirect measure. We see that the difference (3 ktCO_2e) is explained by the 0.97 ktCO_2e for the Allocation Effect and 1.89 ktCO_2e for the Selection Effect. The major contribution of the Allocation Effect is by far the Utilities sector with 0.82 ktCO_2e. This sector represents, on average, 7.79% of the portfolio *Euro* and only 5.24% in the natural benchmark, and usually is associated with very high levels of GHG emissions, as it can be seen in the GHG Emission columns in Table 10.

To be noted, this sector is also responsible for a significant part of the Selection Effect (0.49 ktCO_2e). We may argue that Utility companies in the Euro zone are a significant contributor of the total GHG emissions of the portfolio *Euro*, both because they are big emitters and they represent a significant part of the Euro zone financial market. The Oil & Gas sector also contributes to the portfolio's larger emissions through the Selection Effect: although this sector is under weighted in the portfolio (6.25% versus 8.74%), the selected stocks have indeed higher emissions than the natural benchmark (24.72 ktCO_2e for the sector in the portfolio versus 19.77 ktCO_2e in the natural benchmark). The same is also true for Basic Materials.

For the portfolio *Ex-Euro* we see that its lower GHG emissions are due to the Selection Effect for -2.6 ktCO_2e and the Allocation Effect for -1.02 ktCO_2, as shown in Table 11.

The Utility sector represents half of the GHG emissions reduction: -0.85 ktCO_2e in the Allocation Effect and -1.27 ktCO_2e in the Selection Effect. Indeed the portfolio *Ex-Euro* is under-weighted this sector and the stocks that it selects have lower emissions than the stocks in the same sector selected by the natural benchmark.

A similar conclusion can be drawn for the Basic Material sector: stocks in this sector outside the Euro zone tend to have lower emissions than the natural benchmark.

The finding is then symmetric with respect to the combined effect of these two sectors for the portfolio *Euro*.

In conclusion, we find that the geographic distinction has a clear effect in the GHG emissions of the portfolios: the portfolio *Euro* has higher GHG emissions than the benchmark (and the *Ex-Euro* too), mainly because it usually overweights the Utility sector, and Utility stocks in the Euro zone have higher GHG emission than their peers in the rest of Europe, and the same applies for stocks in the Basic Material sector, although there is no overweighting of this sector for both portfolios.

Euro

Sector	Allocation	Selection	Interaction	Total	Avg. Weight *Euro*	GHG Emissions	Contr.	Avg. Weight Bench.	GHG Emissions	Contr.
Total	0.97	1.89	0.14	3		11.13			8.13	
Basic Materials	0.07	0.39	0	0.46	8.81%	25.09	2.14	8.22%	20.27	1.64
Cons. Goods	0.02	0.18	-0.01	0.19	14.57%	5.92	0.79	15.26%	4.61	0.66
Cons. Services	0.02	0.09	-0.01	0.1	6.69%	6.52	0.43	7.16%	5.34	0.38
Financials	-0.1	0.07	0	-0.03	25.18%	0.93	0.21	23.98%	0.6	0.14
Health Care	0.43	0.02	-0.01	0.44	4.75%	0.63	0.03	10.46%	0.41	0.04
Industrials	0.08	0.19	0.03	0.3	14%	13.65	1.85	11.86%	11.98	1.37
Oil - Gas	-0.19	0.44	-0.09	0.15	6.95%	24.72	1.7	8.74%	19.77	1.7
Technology	-0.13	0	0	-0.13	5.13%	0.73	0.04	3.38%	0.66	0.02
Telecom.	-0.03	0.01	0	-0.02	6.1%	0.88	0.05	5.71%	0.7	0.04
Utilities	0.82	0.49	0.22	1.53	7.79%	53.35	3.89	5.24%	42.46	2.15

Table 10: GHG emission attributions for the portfolio *Euro* over its natural benchmark. Source DataStream, Stoxx, TruCost.

Ex-Euro

Sector	Allocation	Selection	Interaction	Total	Avg. Weight *Ex-Zone*	GHG Emissions	Contr.	Avg. Weight Bench.	GHG Emissions	Contr.
Total	-1.02	-2.6	0.49	-3.14		6.02			9.16	
Basic Materials	-0.08	-0.42	0.01	-0.49	7.75%	17.69	1.33	8.22%	22.9	1.86
Cons. Goods	-0.02	-0.17	-0.01	-0.2	15.97%	3.86	0.6	15.26%	5.02	0.73
Cons. Services	-0.02	-0.08	0	-0.1	7.62%	4.91	0.37	7.16%	6.03	0.43
Financials	0.11	-0.08	0	0.03	22.71%	0.35	0.08	23.98%	0.72	0.16
Health Care	-0.45	-0.01	0	-0.46	15.69%	0.4	0.06	10.46%	0.46	0.05
Industrials	-0.08	-0.29	0.05	-0.32	9.89%	10.67	1.04	11.86%	13.19	1.53
Oil - Gas	0.2	-0.28	-0.06	-0.14	10.41%	18.81	1.87	8.74%	21.95	1.86
Technology	0.13	-0.01	0	0.13	1.77%	0.56	0.01	3.38%	0.74	0.02
Telecom.	0.03	-0.01	0	0.02	5.34%	0.64	0.03	5.71%	0.81	0.05
Utilities	-0.85	-1.27	0.51	-1.61	2.82%	22.41	0.63	5.24%	49.93	2.46

Table 11: GHG emission attributions for the portfolio *Ex-Euro* over its natural benchmark. Source DataStream, Stoxx, TruCost.

6 Conclusions

This paper provides a unified framework for calculating carbon footprint and carbon emissions attributions for dynamically rebalanced funds with respect to their benchmarks. The procedure applies for emissions measures, risk measures (typically measures of coal, oil and gas reserves) and impact measures (typically measures of green energy production or impact investing). This range of indicators can be calculated within a unique and coherent framework, improving then the standardization and helping comparisons between different opportunities and investments.

All together they constitute a complete carbon footprint and give a full picture of investors' portfolios exposures to climate change risk.

Following the literature and standard practice, we distinguish between absolute and relative (intensity) measures, where the latter normalizes absolute measures over total revenues. For both measures, we present carbon performance attributions with respect to the benchmark with which investors can disentangle the excess contribution over the benchmark into an Allocation Effect and a Selection Effect. As for financial performance attributions, these exercises highlights excess contributions across specific categories (typically industrial sectors or countries) and could help investors to redirect their investments (by over/under-weighting given categories or by changing the selection within them) in order to improve their total contribution or their portfolios' intensities.

We provide two examples of carbon footprinting and carbon emissions attribution for European stocks. The first one shows that companies' size is an important factor to take into account. Indeed small companies tend to have higher GHG emissions than their larger counterparts, but not necessarily higher intensities. The second one compares the differences between Euro zone stocks versus stocks in the rest of Europe. As a matter of fact, the geographic factor is also important: Euro zone stocks do have higher GHG emissions than the rest of Europe, and this is mainly explained by the significant role played by Utilities stocks in the Eurozone.

A Alternative carbon attribution for Intensities

This alternative decomposition is similar in the spirit of the Brinson decomposition as it does not use quantities that depend on the fund as global constants. The price to pay is the inclusion of an extra term in the decomposition that now manages the double Revenue/Intensity Allocation Effect. From (3.3) and (4.1) we can write

$$
\begin{aligned}
I^F(X) &= \frac{A^F(X)}{A^F(R)} \\
&= \frac{\sum_{t=T_1}^{T_2}\sum_{k=1}^{|S|} W_t^{S_k} A^{F_{k,t})}(t,t,X)}{A^F(R)} \\
&= \sum_{t=T_1}^{T_2}\sum_{k=1}^{|S|} \frac{W_t^{S_k} A^{F_{k,t}}(t,t,R)}{A^F(R)} \frac{A^{F(k)}(t,t,X)}{A^{F(k)}(t,t,R)} \\
&= \sum_{t=T_1}^{T_2}\sum_{k=1}^{|S|} W_t^{S_k} R_t^{F_{k,t}} I^{F_{k,t}}(t,t,X)
\end{aligned}
$$

where the ratio

$$R_t^{F_{k,t}} := A^{F_{k,t}}(t,t,R)/A^F(R)$$

represents the contribution of sector S_k on day t to the total revenue over the period and $I^{F_{k,t}}(t,t,X)$ is the instantaneous intensity of the fund $F_{k,t}$. The same can be deduced for BF, so that the excess intensity of the fund over the benchmark is

$$EI(X) = I^F(X) - I^{BF}(X) := \sum_{t=T_1}^{T_2} EI_t(X)$$

where

$$EI_t(X) := \sum_{k=1}^{|S|} W_t^{S_k} R_t^{F_{k,t}} I^{F_{k,t}}(t,t,X) - \sum_{k=1}^{|S|} W_t^{B_k} R_t^{BF_{k,t}} I^{BF_{k,t}}(t,t,X)$$

With respect to (4.6), $EI(X)$ does not depend on $A^F(R)$ anymore.

As before, we concentrate on instantaneous excess intensity EI_t and the total EI as the latter is simple the sum of the former over the period in study. Let us define

$$R_t^F = \sum_{k=1}^{|S|} R_t^{F_{k,t}} \quad \text{and} \quad R_t^{BF} = \sum_{k=1}^{|S|} R_t^{BF_{k,t}}$$

which are the proportion of total revenues attributable to the fund F and the natural benchmark BF on day t. To shorten the notations, we denote $I_t^{F_{k,t}} := I^{BF_{k,t}}(t,t,X)$ and $I_t^{F_{k,t}} := I^{F_{k,t}}(t,t,X)$ and we drop, wherever it is not ambiguous, the dependence on X. Then

$$\begin{aligned}
EI_t =& \sum_{k=1}^{|S|} \left(W_t^{S_k} - W_t^{B_k}\right) I_t^{BF} \left(R_t^{BF_{k,t}} - R_t^{BF}\right) && \Leftrightarrow \text{Revenue Allocation Effect} \\
&+ \sum_{k=1}^{|S|} \left(W_t^{S_k} - W_t^{B_k}\right) R_t^{BF} \left(I_t^{BF_{k,t}} - I_t^{BF}\right) && \Leftrightarrow \text{Intensity Allocation Effect} \\
&+ \sum_{k=1}^{|S|} \left(W_t^{S_k} - W_t^{B_k}\right) \left(I_t^{BF_{k,t}} - I_t^{BF}\right) \left(R_t^{BF_{k,t}} - R_t^{BF}\right) && \Leftrightarrow \text{Revenue/Intensity Allocation Effect} \\
&+ \sum_{k=1}^{|S|} W_t^{B_k} \frac{\left(I_t^{F_{k,t}} + I_t^{BF_{k,t}}\right)}{2} \left(R_t^{F_{k,t}} - R_t^{BF_{k,t}}\right) && \Leftrightarrow \text{Revenue Selection Effect} \\
&+ \sum_{k=1}^{|S|} W_t^{B_k} \frac{\left(R_t^{F_{k,t}} + R_t^{BF_{k,t}}\right)}{2} \left(I_t^{F_{k,t}} - I_t^{BF_{k,t}}\right) && \Leftrightarrow \text{Intensity Selection Effect} \\
&+ \sum_{k=1}^{|S|} \left(W_t^{S_k} - W_t^{B_k}\right) \left(I_t^{F_{k,t}} R_t^{F_{k,t}} - I_t^{BF} R_t^{BF}\right) && \Leftrightarrow \text{Interaction Effect}
\end{aligned}$$

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About Ossiam

Ossiam is a research-driven French asset management firm (authorized by the Autorité des Marchés Financiers) and specializes in delivering smart beta* solutions. Efficient indexing is at the core of Ossiam's business model. The firm was founded in response to a post-subprime crisis demand from investors for simplicity, liquidity and transparency. Given the environment, there was a growing need among investors for enhanced beta exposure and risk hedging. Ossiam is focused on the development of innovative investment solutions for investors via a new generation of indices.

*'Smart beta' refers to systematically managed, non-market-cap-weighted strategies covering any asset class.

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